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PLANNING AND CONTROL UNDER RISK

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ABSTRACT

A variety of different research efforts have been supported in the past three years. This research falls in the following areas:

(1) Theory and computation of optimal policies in dynamic programming risk problems.

(2) Applied stochastic processes.

(3) Development of models for institutional operating policies.

(4) Linearized Bayesian estimation models.

A summary of the research effort in each of the above areas is presented.

PLANNING AND CONTROL UNDER RISK

A variety of different research efforts have been supported during the past three years.

One main area of investigation has been that of the theory and computation of dynamic programming including Markov and semi-Markov decision problems. For instance, in [73-17] a model of optimal allocation under risk was developed; the problem is to build an n component system which is said to function if at least k (out of n) components function. To add a component we must first decide how much money to allocate to that task; when x dollars are invested in a component then the component will function with probability $P(x)$. Given a total budget of A dollars the problem of interest is to determine how much money should be invested in each component so as to maximize the probability of attaining a functioning system. The problem is considered both in the sequential and nonsequential case, and conditions under which it is optimal to invest A/n dollars for each of n components are presented. The special case where $P(x) = \min(x,1)$ is considered in some detail. For this case the optimal strategy is determined in the sequential case when $k = 2$ and a conjecture is made in the case of arbitrary k.

In order to obtain some insight into the structure of optimal policies in risk models, a class of gambling models, useful as simple prototypes for risk models, was considered in [72-24]. For a variety of objectives, it was shown that if the game is favorable to the player, then he should play as timidly as possible; that is, always make the smallest bet. A model in which the gambler is also given the option of working is considered, and it is shown that if the available gambles are unfavorable then the strategy which minimizes the gambler's expected time to reach some preassigned goal is the strategy that always calls for working. For the same model it is also shown that if the

work option is only available at certain times (namely, when the gambler is broke) then the optimal gambling strategy is to play boldly. These results were obtained by developing some new general results in dynamic programming, also given in [72-24]. Other applications, such as determining the optimal customer selection in exponential queues [71-24], and determining the optimal strategy for buyers and sellers of stock options [74-1], have also been considered.

Related research in the theory of applied stochastic processes have dealt with bounds on the delay distribution in single server queueing models [73-1] and a study of the maximum value of the continuous time version of a random walk process [71-29]. Recent work has considered the study of a multicomponent reliability system in which each individual component is either in a working or a failed condition. It is supposed that when a component is in its failed condition then a repair is initiated which takes a random length of time. For an arbitrary system structure (i.e., series, parallel, k-out-of-n, etc.) such quantities as the average system failure rate, and the average system uptimes and downtimes were derived [74-4]. In addition in [74-8] it was shown that when all component distributions are exponential then the time to first failure has the NBU (i.e., new better than used) property. This and other results were then used to obtain a lower bound to the mean time until first system failure.

Another major area supported has been the development of models of institutional staffing and educational system analysis with emphasis on equilibrium flow prediction, and evaluation of instructional costs under different operating policies [71-16, 71-18].

Many decision models require Bayesian updating of prior information in order to incorporate experience data into the optimization. Recent research into linearized Bayesian forecasting methods, called "credibility theory" has proved to be very productive, leading to many new and interesting linear

formulas [73-17, 73-13]. The importance of this linearized technique is twofold. In the first place the exact Bayesian estimator (usually the conditional expectation of the unknown parameter given all accumulated data) is, in practice, usually quite difficult to compute, whereas the computation of the first linear estimate is usually straightforward. Secondly, the best linear estimate can usually be computed with only a knowledge of some of the parameters of the prior (usually the mean and variance) and thus can be obtained even when the prior distribution is not completely specified. Also, it can be shown that, in many models, the credibility result is exact Bayesian or an excellent approximation. In particular, in 73-21, it is shown that the best linear approximation is identical to the optimal decision rule when the probability distribution and its conjugate prior are members of the exponential family of distribution. Also, in work related to this a method of approximating the optimal decision rule by the use of orthogonal functions is presented in [73-24].

REFERENCES

[71-16] Oliver, R. M. and D.S.P. Hopkins, "Instructional Costs of University Outputs," (July 1971).

[71-18] Oliver, R. M. and D.S.P. Hopkins, "An Equilibrium Flow Model of a University Campus," (July 1971).

[71-24] Cramer, M., "Optimal Customer Selection in Exponential Queues," (September 1971).

[71-29] Ross, S. M., "On the Maximum of a Stationary Independent Increment Process," (November 1971).

[72-24] Ross, S. M., "Dynamic Programming and Gambling Models," (September 1972).

[73-1] Ross, S. M., "Bounds on the Delay Distribution in GI/G/1 Queues," (January 1973).

[73-7] Jewell, W. S., "Multi-Dimensional Credibility," (April 1973).

[73-13] Jewell, W. S., "The Credible Distribution," (August 1973).

[73-17] Ross, S. M., C. Derman and G. J. Lieberman, "Optimal Allocations in the Construction of k-Out-of-n Reliability Systems," (September 1973).

[73-21] Jewell, W. S., "Credible Means Are Exact Bayesian for Exponential Families," (October 1973).

[73-24] Pechlivanides, P. M., "Approximations by Orthogonal Functions in Casualty Insurance," (October 1973).

[74-1] Pasternack, B. A., "Optimal Gambling and Investment Systems Under Discounting and Disbursement," (January 1974).

[74-4] Ross, S. M., "Multicomponent Reliability Systems," (February 1974).

[74-8] Ross, S. M., "On Time to First Failure in Multicomponent Exponential Reliability Systems," (March 1974).

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Oliver, R. M. and D.S.P. Hopkins, "Instructional Costs of University Outputs," ORC 71-16, July 1971.

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Ross, S. M., "Bounds on the Delay Distribution in GI/G/1 Queues," ORC 73-1, January 1973.

Salmond, D., "Premium Calculation in Casualty Insurance," ORC 73-2, February 1973.

Symonds, G. H., "A Model for Socioeconomic Studies," ORC 73-4, February 1973.

Jewell, W. S., "Multi-Dimensional Credibility," ORC 73-7, April 1973.

*Barnett, D. D., "A Generalized Gradient in Optimal Control," ORC 73-12, July 1973.

Ross, S. M., C. Derman and G. J. Lieberman, "Optimal Allocations in the Construction of k-Out-of-n Reliability Systems," ORC 73-17, September 1973.

Jewell, W. S., "The Credible Distribution," ORC 73-13, August 1973.

Jewell, W. S., "Credible Means Are Exact Bayesian for Exponential Families," ORC 73-21, October 1973.

Pechlivanides, P. M., "Approximations by Orthogonal Functions in Casualty Insurance," ORC 73-24, October 1973.

*Arthur, W. B., "Optimal Control Theory with Time Delay," ORC 73-27, November 1973.

*Pasternack, B. A., "Optimal Gambling and Investment Systems Under Discounting and Disbursement," ORC 74-1, January 1974.

Ross, S. M., "Multicomponent Reliability Systems," ORC 74-4, February 1974.

Ross, S. M., "On Time to First Failure in Multicomponent Exponential Reliability Systems," ORC 74-8, March 1974.

Jewell, W. S., "Exact Multidimensional Credibility," ORC 74-14, May 1974.

*Rath, A., "Identification of a Distributed Parameter System in Hydrology," ORC 74-15, May 1974.

Jewell, W. S., "Isotonic Optimization and Tariff Construction," ORC 74-20, July 1974.

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